**One problem with many solutions.**
https://www.linkedin.com/feed/update/urn:li:activity:6519146412557631488
Let $a,b,x,y$ be real numbers such that $ay - bx = 1$.
Prove that $a^2+b^2+x^2+y^2+ax+by \geq \sqrt{3}$.
**Solution by Arkady Alt, San Jose, California, USA.**
Let $a = p\cdot\cos\alpha, b := p\cdot\sin\alpha, x := r\cdot\cos\varphi, y := r\cdot\sin\varphi$, where $p,r>0$ and $\alpha,\varphi$ be any.
Then $ay - bx = 1 \Leftrightarrow pr\cdot\sin(\varphi-\alpha) = 1$ and

$$a^2+b^2+x^2+y^2+ax+by = p^2+r^2+pr\cdot\cos(\varphi-\alpha).$$

Let $t := \varphi-\alpha$. Since $\sin t = \dfrac{1}{pr} > 0$ and $p^2+r^2+pr\cdot\cos(\varphi-\alpha) =$
$p^2+r^2+pr\cdot\cos t \geq 2pr - pr|\cos t| = \dfrac{1}{\sin t}(2-|\cos t|)$ then suffice to prove inequality

$$\frac{1}{\sin t}(2-\cos t) \geq \sqrt{3} \text{ for } t \in (0,\pi/2).$$

We have $\dfrac{1}{\sin t}(2-\cos t) \geq \sqrt{3} \Leftrightarrow 2 \geq \sqrt{3}\sin t + \cos t \Leftrightarrow \sin\left(t+\dfrac{\pi}{3}\right) \leq 1.$
**And more solutions.**
**Solution 2.**
Let $p := a^2+b^2+x^2+y^2, q := (a^2+b^2)(x^2+y^2) = (ax+by)^2+1 \geq 1.$
Since $p \geq 2\sqrt{q}$ then $a^2+b^2+x^2+y^2+ax+by \geq a^2+b^2+x^2+y^2-|ax+by| =$
$p-\sqrt{q-1} \geq 2\sqrt{q}-\sqrt{q-1}$ and further possible three different endings:
**Ending 1.**
$2\sqrt{q}-\sqrt{q-1} \geq \sqrt{3} \Leftrightarrow 2 \geq \sqrt{3}\cdot\sqrt{\dfrac{1}{q}}+\sqrt{1-\dfrac{1}{q}}$, where latter inequality is
Cauchy Inequality for $\left(\sqrt{3},1\right), \left(\sqrt{\dfrac{1}{q}},\sqrt{1-\dfrac{1}{q}}\right)$.
**Ending 2.**
Applying inequality $ab-cd \geq \sqrt{a^2-c^2}\cdot\sqrt{b^2-d^2},\ a \geq c > 0, b \geq d > 0$
to $(a,c) = (2,1)$ and $(b,d) = \left(\sqrt{q},\sqrt{q-1}\right)$ we obtain $2\sqrt{q}-\sqrt{q-1} \geq \sqrt{3}$.
**Ending 3.**
Since $q \geq 1$ then, denoting $t := \arccos\dfrac{1}{\sqrt{q}} \in [0,\pi/2)$ we obtain

$$2\sqrt{q}-\sqrt{q-1}-\sqrt{3} = \frac{2}{\cos t}-\sqrt{\frac{1}{\cos^2 t}-1}-\sqrt{3} = \frac{2}{\cos t}-\tan t-\sqrt{3} =$$
$$\frac{2\left(1-\sin\left(\frac{\pi}{3}+t\right)\right)}{\cos t} \geq 0.$$

**Solution 3.**
Let $u := a^2+b^2, v := x^2+y^2, w := ax+by$ and let $t := u+v+w$. First note that
$t>0$. Indeed, since $u+v \geq 2|ax|+2|bx|$ then $t \geq |ax|+|bx|+(|ax|+ax)+(|bx|+bx) \geq 0$.
Since Vieta's system $u+v = w-t$ and $uv = w^2+1$ solvable in real $u,v,w$ iff
$(w-t)^2 \geq 4(w^2+1) \Leftrightarrow 3w^2+2tw-t^2+4 \leq 0$ then $t \geq 0$ should provide
nonnegativity of discriminant of quadratic trinomial $3w^2+2tw-(t^2-4)$
that is $t^2+3t^2-12 \geq 0 \Leftrightarrow t \geq \sqrt{3}$.

**And in addition to solutions presented above**:

**Solution of Giovanni Parzanese**

Let $u := a^2 + b^2, v := x^2 + y^2, w := ax + by.$ In such notation and since $ay - bx = 1$ Lagrange Identity $(a^2 + b^2)(x^2 + y^2) = (ax + by)^2 + (ay - bx)^2$ becomes $uv = w^2 + 1$

Then $a^2 + b^2 + x^2 + y^2 + ax + by = u + v + w = \frac{uv + v^2 + vw}{v} = \frac{w^2 + 1 + v^2 + vw}{v}$

and since $\frac{w^2 + v^2 + vw}{v} \geq \frac{3v}{4} \Leftrightarrow (v + 2w)^2$ we obtain $\frac{w^2 + 1 + v^2 + vw}{v} =$

$\frac{w^2 + v^2 + vw}{v} + \frac{1}{v} \geq \frac{3v}{4} + \frac{1}{v} \geq 2\sqrt{\frac{3v}{4} \cdot \frac{1}{v}} = \sqrt{3}.$

**Remark**

In my opinion it is ideal solution of the problem. I mean representation

$U + W + V = \frac{UW + W^2 + VW}{W} = \frac{V^2 + W^2 + VW}{W} + \frac{1}{W}$ and using inequality

$\frac{V^2 + W^2 + VW}{W} \geq \frac{3W}{4}$